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NUMERICAL SIMULATION OF PHOTOVOLTAIC GENERATORS WITH SHADED CELLS

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ABSTRACT

A model which is able to describe the relationship between current and voltage of a solar cell in generation region as well as the breakdowns at positive and negative voltages is given. A set of suitable numerical algorithms to compute the currents for a given voltage at the solar cell and vice versa is described. A general model for the description of solar generators is proposed which gives all voltages and currents as well as the voltage and current at the output of the solar generator itself. This new model is also implemented in a numerical algorithm.

1. INTRODUCTION

For the description of the electrical behaviour of a solar cell during generating operation the two diodes model is commonly used, see e.g. Overstraeten [5].

For cells which are driven in the negative voltage range another model is necessary, which describes the breakdown region at high negative voltages. Negative voltages at solar cells can occur at non-uniform illuminated photovoltaic generators, especially during partially shading of the generator. One model was described by Rauschenbach [9]. A more precise model, based on the one diode model was given by Bishop [2]. This model can be adapted to the two diodes model and offers optimal conditions for the description of the solar cell characteristics.

Several simulation programs for photovoltaic generators were developed during the last two decades. Owing to the development of more powerful computers, they became more complex and more efficient. However, these programs still offer limited possibilities to simulate the electrical behaviour of photovoltaic generators under mismatched conditions or partial shading. For this purpose, a more general electrical model which allows simulations at all conditions has been developed. Numerical methods, such as described by Press at. al. [6] are used to obtain a solution for this model.

2. MODEL OF A SOLAR CELL

The two diodes model is widely used for an equivalent circuit. Shaded cells of a non-uniform illuminated solar module or solar generator can be driven into the negative voltage region. If there are no bypass diodes for cell protection, a diode breakdown can happen during high negative voltages. This breakdown is not taken into account in the two diodes model. Therefore another model based on the model of Bishop is described in this paper. This model includes an extension term which describes the diode breakdown at high negative voltages. The equivalent of this model is given in figure 1.

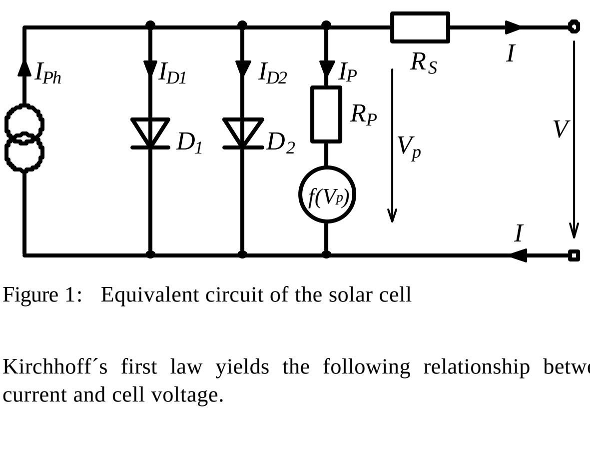

Figure 1: Equivalent circuit of the solar cell

Kirchhoff´s first law yields the following relationship between cell current and cell voltage.

$$0 = f(V,I) = I_{Ph} - I_{D1} - I_{D2} - I_P - I = I_{Ph} - I_{S1} \cdot \left(\exp\left(\frac{V + I \cdot R_S}{m_1 \cdot V_T} \right) - 1 \right) - I_{S2} \cdot \left(\exp\left(\frac{V + I \cdot R_S}{m_2 \cdot V_T} \right) - 1 \right) - \frac{V + I \cdot R_S}{R_P} \underbrace{- a \cdot \frac{V + I \cdot R_S}{R_P} \cdot \left(1 - \frac{V + I \cdot R_S}{V_{Br}} \right)^{-n}}_{\text{Extension term for the negative dio de breakdown}} - I \tag{1}$$

This equation differs from the usual two diodes model by the extension term marked in the equation above. The electrical behaviour of the solar cell can be described by this equation over the whole voltage range.

3. SIMULATION OF A SOLAR CELL

In order to model the solar cell curve, the current *I*, for a given voltage *V*, must be computed individually for each value. Because the equation of the solar cell curve is not given in explicit form, numerical methods are normally used to determine the characteristic curve.

An equation for the solar cell $f(V,I) = 0$ is given in equation (1). For a given voltage V the current I is given by the root of equation (1), where I is expected to be the single root.

In order to determine the single root, it is suggested to use Newton-Raphson´s method, described e.g. by Mathews [4], as it offers considerable advantages over other methods. The main ideas of this algorithm will be described shortly. Starting with the initial value I_0, the following iteration is executed:

$$I_{i+1} = I_i - \frac{f(V, I_i)}{\frac{\partial f(V, I_i)}{\partial}} \tag{2}$$

The iteration stops if a suitable condition is met. In the case above the absolute difference of two succeeding values of the iteration is used:

$$\left| I_{i+1} - I_i \right| < \varepsilon .$$

The derivative of the solar cell equation can be given as shown below:

$$\frac{\partial f(V,I)}{\partial} = - \frac{I_{S1} \cdot R_S}{m_1 \cdot V_T} \cdot \exp\left(\frac{V + I \cdot R_S}{m_1 \cdot V_T} \right) - \frac{I_{S2} \cdot R_S}{m_2 \cdot V_T} \cdot \exp\left(\frac{V + I \cdot R_S}{m_2 \cdot V_T} \right) - \frac{R_S}{R_P} - 1 - a \cdot \frac{R_S}{R_P} \cdot \left(1 - \frac{n \cdot (V + I \cdot R_S)}{V + I \cdot R_S - V_{Br}} \right) \cdot \left(1 - \frac{V + I \cdot R_S}{V_{Br}} \right)^{-n} \tag{3}$$

The Newton-Raphson algorithm can be implemented in a computer program easily. An implementation in the C programming language can be written as:

```
float I_Newton (float V)
{     float I1,I2,D, eps = 1e-5;

      I1=IStart;  error=0;
      do {   D=df(V,I1);
             if (D==0) { error=1; return (0); }
             I2 = I1 - f(V,I1) / D;
      } while (fabs (I2-I1) > eps);
      return (I2);
}
```

The Newton-Raphson algorithm has the advantage of a very quick, quadratic convergence for initial values near the root, so that a good solution can be calculated within a few iteration steps. In the proximity of the diode breakdowns difficulties occur. In this region the Newton-Raphson algorithm converges very slowly and can even diverge for badly chosen starting points.
To speed up the convergence, in this case, the bisection method can be used to determine an initial value for the Newton-Raphson method. Then, a more exact solution can be quickly calculated by the Newton-Raphson method. The bisection method converges only linearly, but it cannot fail if the root is in the chosen interval. The interval is bisected and the part of the interval, which does not contain the root is eliminated until the wished exactness is reached. An example for an implementation in the C programming language is also given here.

```
float I_Intervall (float V)
{    float I, delta, eps=0.2;
     I=IStart;     delta=deltaStart;
     do  {  D=f(V,I);
          if (D>0)  I = I + delta;
          if    (D<0)  I = I - delta;
          delta = delta/2;
     } while (delta>eps);
     return (I);
}
```

Figure 2 shows a typical current-voltage characteristic of an unilluminated solar cell over a wide range. The algorithms proposed above coincide with the measured points, proving to be an excellent method to describe the solar cell.

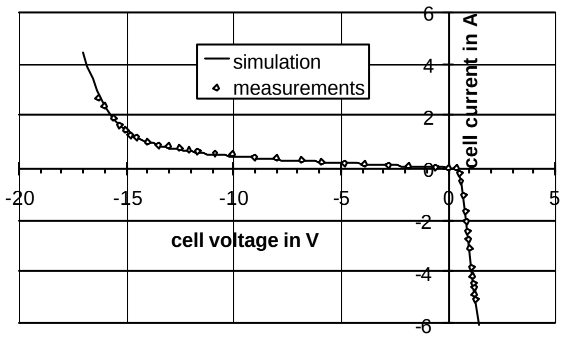

Figure 2: Comparison between measurements and simulation of the characteristic curve of an unilluminated solar cell

4. MODEL OF A PHOTOVOLTAIC GENERATOR

In the previous section the calculation of the characteristics of a solar cell was shown. Now, this model can be used for various interconnections between solar cells, diodes, cables and other components of solar generators.
In this section a method is described for the calculation of large networks. Additionally, the characteristic curve of each component will be determined for each point of the generator curve. This is done to access the power losses of a partly or fully shaded solar cell.
Kirchhoff´s laws are applied to provide an equation system relating all currents and voltages in the network. A continuously varying system of nodes and meshes is difficult to describe for a computer. This is why a general model for an element interconnection is described as shown in figure 3.
The elements can be placed vertically ($A_{i,j}$) or horizontally ($B_{i,j}$) on a regular grid. Altogether there are *z* elements.

The following components can be used as elements.

- solar cell (two diodes equivalent circuit including negative diode breakdown description term)
- diode
- resistance
- ideal conductor (in this case $R_{cond.} = 10^{-20}\Omega$)
- insulation resistance (in this case $R_{insul.} = 10^{20}\Omega$)

etc.

It is also possible to use a whole assembly instead of a single component, for example a string of multiple solar cells. In this case the current-voltage-curve and the derivative of the curve must be known.
Each element is described by the mathematical relationship between current and voltage. These equations can be given in the following form.

$$f(V_{Ai,j}, I_{Ai,j}) = 0 \quad \text{and} \quad f(V_{Bi,j}, I_{Bi,j}) = 0 \,.$$

In this case the current for a given voltage has to be determined by a numerical method. For many elements (e.g. resistance) there is an explicit equation for the current.

$$I_{Ai,j} = g(V_{Ai,j}) \qquad \text{and} \qquad I_{Bi,j} = g(V_{Bi,j}) \,.$$

5. SIMULATION OF A PHOTOVOLTAIC GENERATOR

The total voltage *V* must be given for the determination of all characteristic curves. The relating total current *I*, all sub-voltages and sub-currents, can be calculated for this total voltage. This means, the total voltage *V* can be interpreted as a known quantity and is kept constant for all calculations. For each sub-voltage the relating sub-current can be determined by the relations indicated above, so that the calculation of the sub-voltages is sufficient. Now, there are $z + 1$ unknowns. These are *z* sub-voltages of each element plus the total current *I*.
The vector $\mathbf{u}$ is the vector of the unknown sub-voltages and the unknown total current:

$$\mathbf{u}^{\mathbf{T}} = (V_{A1,1}, \ldots, V_{Ay,x}, V_{B1,1}, \ldots, V_{By-1,x-1}, I) \,.$$

The total voltage is given, for example, by the following equation:

$$r_1(\mathbf{u}) = \sum_{i=1}^{y} V_{Ai,1} - V = 0 \,. \tag{4}$$

In the circuit there are altogether *m* meshes $M_{i,j}$, according to Kirchhoff´s second law. The sum of the voltages in each mesh equals zero.

$$r_{2..m+1}(\mathbf{u}) = V_{Ai,j} - V_{Bi,j} - V_{Ai,j+1} + V_{Bi-1,j} = 0 \tag{5}$$

for i = *1*..y*;* j = *1*..x - *1*
with $V_{Bi,j} = 0$ if *i* = 0 or *i* = *y*

For the total current *I* results the equation:

$$r_{m+2}(\mathbf{u}) = \sum_{j=1}^{x} I_{A1,j} - I = 0 \,. \tag{6}$$

Altogether there are *n* nodes $N_{i,j}$ in the circuit. According to Kirchhoff´s first law, the sum of the currents equals zero.

$$r_{m+3..m+n+2}(\mathbf{u}) = I_{Ai,j} + I_{Bi,j} - I_{Ai+1,j} - I_{Bi,j-1} = 0 \tag{7}$$

for i = *1*..y-*1;* j = *1*..x

For the $z + 1$ unknowns, represented by the vector $\mathbf{u}$, $z + 1$ independent equations $r_1(\mathbf{u})...r_{m+n+2}(\mathbf{u})$ are given. These equations form the non-linear equation system $\mathbf{r}(\mathbf{u})$.

$$\mathbf{r}(\mathbf{u}) = \begin{bmatrix} r_1(\mathbf{u}) \\ r_2(\mathbf{u}) \\ \vdots \\ r_{m+1}(\mathbf{u}) \\ r_{m+2}(\mathbf{u}) \\ r_{m+3}(\mathbf{u}) \\ \vdots \\ r_{m+n+2}(\mathbf{u}) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{y} V_{Ai,1} - V \\ V_{A1,1} - V_{B1,1} - V_{A1,2} \\ \vdots \\ V_{Ay,x-1} - V_{Ay,x} + V_{By-1,x-1} \\ \sum_{j=1}^{x} I_{A1,j} - I \\ I_{A1,1} + I_{B1,1} - I_{A2,1} \\ \vdots \\ I_{Ay-1,x} - I_{Ay,x} - I_{By-1,x-1} \end{bmatrix} \tag{8}$$

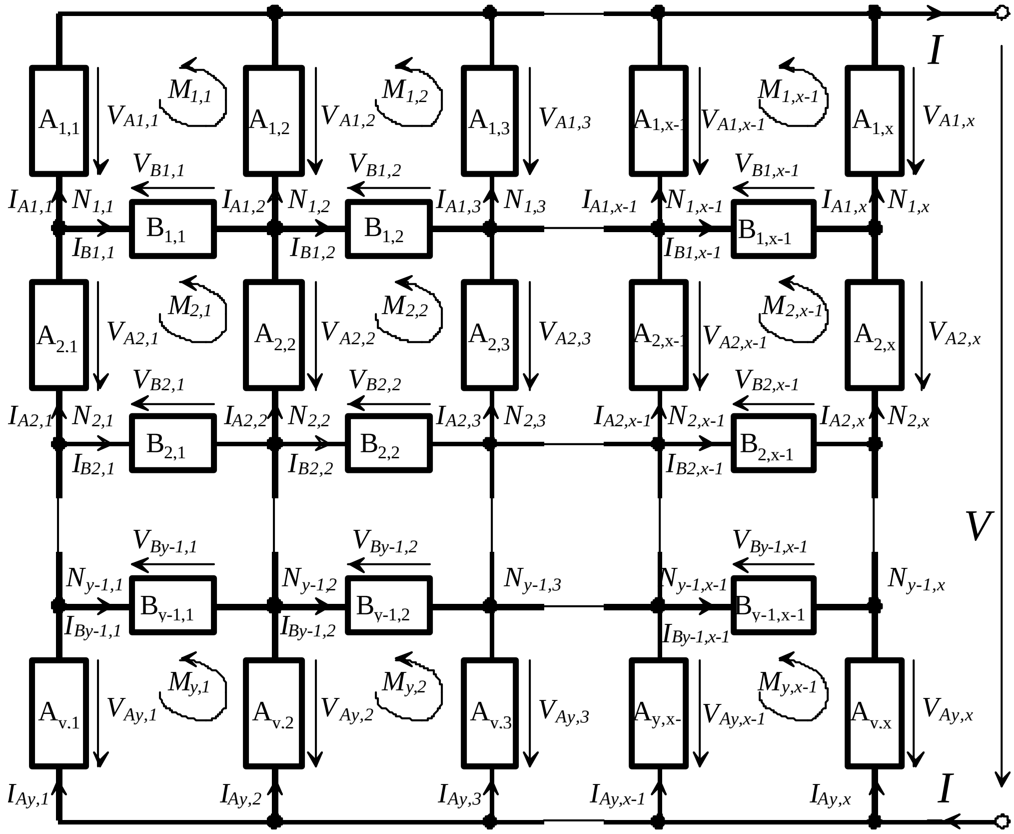

Figure 3: General model for a connection of various elements in a photovoltaic assembly

When using the solution vector $\mathbf{u}$ for a given total voltage V, all results of the equations of this equation system should be zero ($r_1(\mathbf{u}) = r_2(\mathbf{u}) = \ldots\ r_{m+n+2}(\mathbf{u}) = 0$). The root of the equation system, represented by the vector $\mathbf{u}$, which satisfies the equation $\mathbf{r}(\mathbf{u}) = \mathbf{0}$, is to be found.

This equation system shall also be solved by applying numerical methods. The Newton-Raphson iteration requirement of a single equation with one unknown can be widened on the equation system. This results in

$$\mathbf{u}_{i+1} = \mathbf{u}_i - \left(\mathbf{J}(\mathbf{u}_i)\right)^{-1}\mathbf{r}(\mathbf{u}_i)\ . \qquad (9)$$

Beginning with the starting vector $\mathbf{u}_0$, this iteration should be performed until the stopping condition $\|\mathbf{u}_{i+1} - \mathbf{u}_i\| < \varepsilon$ is satisfied, ε having previously been determined.

The vector $\mathbf{u}_{i+1}$, which satisfies the stopping condition, is also the solution vector and includes the searched sub-voltages and the total current I.

The matrix $\mathbf{J}(\mathbf{u}_i)$ is called Jacobian matrix. The inverse of the Jacobian matrix $(\mathbf{J}(\mathbf{u}_i))^{-1}$ has to be determined for each iteration. The matrix is defined as shown below:

$$\mathbf{J}(\mathbf{u}_i) = \begin{bmatrix} \frac{\partial r_1(\mathbf{u}_i)}{\partial u_1} & \cdots & \frac{\partial r_1(\mathbf{u}_i)}{\partial u_{z+1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial r_{z+1}(\mathbf{u}_i)}{\partial u_1} & \cdots & \frac{\partial r_{z+1}(\mathbf{u}_i)}{\partial u_{z+1}} \end{bmatrix} = $$

$$= \begin{bmatrix} \frac{\partial r_1(\mathbf{u}_i)}{\partial V_{A1,1}} & \cdots & \frac{\partial r_1(\mathbf{u}_i)}{\partial I} \\ \vdots & \ddots & \vdots \\ \frac{\partial r_{z+1}(\mathbf{u}_i)}{\partial V_{A1,1}} & \cdots & \frac{\partial r_{z+1}(\mathbf{u}_i)}{\partial I} \end{bmatrix} \qquad (10)$$

The partial derivatives of the mesh equations always equal ±1 or 0. The derivatives of the node equations are more difficult to determine, especially if the current-voltage equations are not given in an explicit form. The derivatives, needed for the Jacobian matrix, can be calculated using the following expression:

$$\frac{dI(V)}{dV} = -\frac{\frac{\partial f(V,I)}{\partial V}}{\frac{\partial f(V,I)}{\partial I}} \qquad (11)$$

The Newton-Raphson method for equation systems shows similar features to those of the Newton-Raphson method for single equations and can also diverge at bad conditioned starting vectors and may obtain meaning-less values. To avoid divergence problems the Newton-Raphson algorithm can be weakened by the following equation which was formulated by Press et. al. [6]. However, this iteration converges more slowly than the original Newton algorithm.

$$\mathbf{u}_{i+1} = \mathbf{u}_i - \lambda \cdot \left(\mathbf{J}(\mathbf{u}_i)\right)^{-1}\mathbf{r}(\mathbf{u}_i) \qquad 0 < \lambda \leq 1 \qquad (12)$$

The total characteristic curve can be determined by giving the total voltage V and calculating the total current I case by case using the iteration given above.

If sufficient values are calculated by the variation of the voltage V, the total curve can be described. The voltage range each element runs through is also obtained.

The described method allows as well to determine all sub-voltages and the total voltage V for a given total current I. For this purpose, the current I in the vector $\mathbf{u}$ must be replaced by the total voltage V. In this case several partial derivatives in the Jacobian matrix are changed.

Figure 4 shows a current-voltage characteristic of a photovoltaic module with 36 solar cells without bypass diodes at two different illumination situations. One curve shows the current-voltage characteristic of a uniform illuminated solar module at an irradiance of E=407W/m² and a

temperature T=300K. The other curve shows the current-voltage characteristic of the same module at the same irradiance and temperature, but 75% of one solar cell of the module is shaded. The simulated curve of the photovoltaic module coincides just as well as the simulated curve of the single solar cell in section 3 with the measured points. Therefore, the described method for the photovoltaic generator is perfectly suited to obtain simulations for the current-voltage characteristics.
The influence of module shading on the module performance can be also seen in Figure 4. The performance loss is 70% although only 2% of the module area are shaded. Most of the performance difference has to be dissipated by the shaded cell. To avoid cell damaging bypass diodes over cell strings of 16 up to 20 cells are integrated in the module, but the performance loss can be only unessentialy reduced by these diodes. A separate bypass diode over every cell can minimise the performance loss. Therefore, Green [3] proposed to integrate bypass diodes into solar cells.

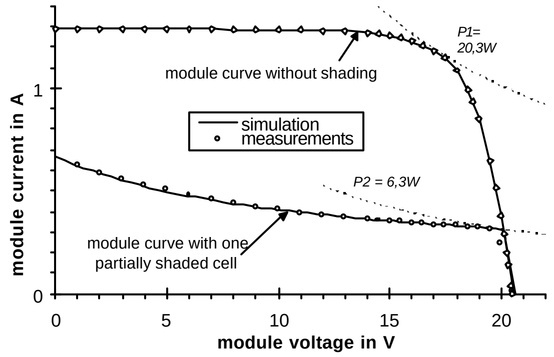

Figure 4: Comparison between measurements and simulation of the characteristic curve of a photovoltaic module with 36 cells without bypass diodes

6. CONCLUSIONS

A model for the description of the total characteristic curve for a solar cell, based on the two diodes model and the model of Bishop [2], is described. Numerical methods are necessary to determine the current voltage curve. Some well-known numerical algorithms and their implementation are described. Furthermore, a general model for the determination of a solar generator curve as well as all sub-voltages and sub-currents of the solar generator is given. The suitable algorithms for the solution of the equation system are introduced for this model.
By analysing the sub-curves, statements regarding possible power losses at mismatched or shaded solar cells can be made. This is necessary to minimise the power losses resulting from shading, and to avoid the damaging of solar cells by hot spots during high power losses.

NOMENCLATURE

e — accuracy for stopping iteration
λ — factor in the modified Newton iteration
a — correction factor (a = 0..1)
$A_{i,j}$ — vertical element (i, j running position index)
$B_{i,j}$ — horizontal element (i, j running position index)
f — function
i — index
I — current of the solar cell or the solar generator
$I_{Ai,j}$ — current at element $A_{i,j}$
$I_{Bi,j}$ — current at element $B_{i,j}$
I_{Ph} — photo current
I_{S1} — saturation current of the first diode
I_{S2} — saturation current of the second diode
j — index
$\mathbf{J}(\mathbf{u})$ — Jacobian matrix
m — number of meshes; $m = y \cdot (x - 1)$
m_1 — ideality factor of the first diode
m_2 — ideality factor of the second diode
$M_{i,j}$ — Kirchhoff mesh (i, j running position index)
n — avalanche breakdown exponent (n = 1..10)
n — number of nodes; $n = x \cdot (y - 1)$
$N_{i,j}$ — Kirchhoff node (i, j running position index)
$\mathbf{r}(\mathbf{u})$ — system of non-linear equations
$r_i(\mathbf{u})$ — partial function of $\mathbf{r}(\mathbf{u})$
R_P — parallel or shunt resistance
R_S — series resistance
$\mathbf{u}$ — vector of unknowns
$\mathbf{u}_0$ — starting vector for the iteration
u_i — element of the vector $\mathbf{u}$
V — voltage of the solar cell or the solar generator
$V_{Ai,j}$ — voltage at element $A_{i,j}$
$V_{Bi,j}$ — voltage at element $B_{i,j}$
V_{Br} — breakdown voltage ($V_{Br} \approx$ -15V..-40V)
V_T — temperature voltage
x — number of horizontal elements of a row
y — number of vertical elements of a column
z — number of all elements (vertical and horizontal);
$z = 2 \cdot x \cdot y - x - y + 1 = m + n + 1$

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